Sparse Linear Prediction and Its Applications to Speech Processing

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Introduction

• Typically the prediction coefficients are found such that the norm-2 of the residual is minimized
  → Maximum likelihood approach when the excitation is considered to be white Gaussian and identically distributed

• Problems
  → Excitation is not always Gaussian (ex. for voiced speech excitation is best represented by a pulse train)
  → Residual not easy to quantize

• Our idea is to use a linear prediction scheme that leaves a sparse residual rather than a minimum variance one
  → More efficient quantization!
Fundamentals

• Mathematically we can state the class of problems as those covered by the optimization problem:
  → Finding the prediction coefficient vector given set of observed real samples

\[
x(n) = \sum_{k=1}^{K} a_k x(n-k) + e(n)
\]

\[
\min_a \left\| x - Xa \right\|_p + \gamma \left\| a \right\|_k
\]

(ex. \( p = 2 \land \gamma = 0 \) → standard LP, autocovariance method)

• Sparseness usually measured using the cardinality, which results in intractable (NP-hard) problems.
  → Instead we use the more tractable Norm-1

(ex. \( p = 1 \land \gamma = 0 \) → ML for laplacian excitation)

• \( \gamma \) can have different interpretation:
  → Regularization term including prior knowledge about the coefficients
  → Minimization interpretation where gamma operates as a Lagrange multiplier
Some Properties of Norm-1 LP Analysis

- Less influenced by outliers  
  \( \rightarrow \text{good for impulse train estimation} \)

- Stability NOT guaranteed  
  \( \rightarrow \text{Use of sub-optimal stable methods (BURG)} \)  
  \( \rightarrow \text{Other tricks (bandwidth expansion, poles reflection)} \)

- Non-uniqueness of the solution (there may be a stable solution in the set…)

- Easily solved using standard linear programming  
  (or convex programming for alternative formulations)
Example of Applications: 
Joint Short & Long Term Linear Prediction

Considering the minimization problem: 

$$\min_a \| x - Xa \|_p^p + \gamma \| a \|_k^k$$

With:  \( p = 2, k = 1, \gamma > 0 \)

Using a high prediction order (ex. 30-40) and a high number of samples (300-400 @8KHz)

We will have a sparse coefficient vector (50-60% null) \( \rightarrow \) factorizable in STLP and LTLP

$$\frac{1}{A_{lp}(z)} \cdot \frac{1}{1 - g_p z^{-T_p}} \approx \frac{1}{A_{slp}(z)}$$
Example of Applications:
Joint Short & Long Term Linear Prediction

\[
\min \frac{1}{a} \left\| x - \frac{Xa}{\|Xa\|_2} \right\|_2^2 + \gamma \left\| a \right\|_1
\]

\[
\min \frac{1}{a} \left\| x - \frac{Xa}{\|Xa\|_2} \right\|_2^2
\]
Conclusions

- Sparse Linear Prediction based on Convex Optimization can be a breakthrough in Speech Coding
  - Residual adapted for the quantizer, rather than the other way around
  - Takes into account statistical properties ignored by the usual LP

- Main drawbacks:
  - Computational load still a bit heavy
    (1 Norm-1 minimization problem ~ 20-30 LS problems!)
  - Stability NOT guaranteed

- It's still a “work in progress” but it gave interesting results so far...
Main References


