A Sparse Nonuniformly Partitioned Multidelay Filter for Acoustic Echo Cancellation

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Motivation

- Nonuniformly partitioned MDF:
  - generalization of uniform partitioning,
  - low algorithmic delay without sacrificing the high order of the adaptive filter (1).
- Sparse constraint on the MDF partitions
  - determines active and inactive regions of the adaptive filter (2,3),
  - updates only coefficients that contribute to the system identification (4).

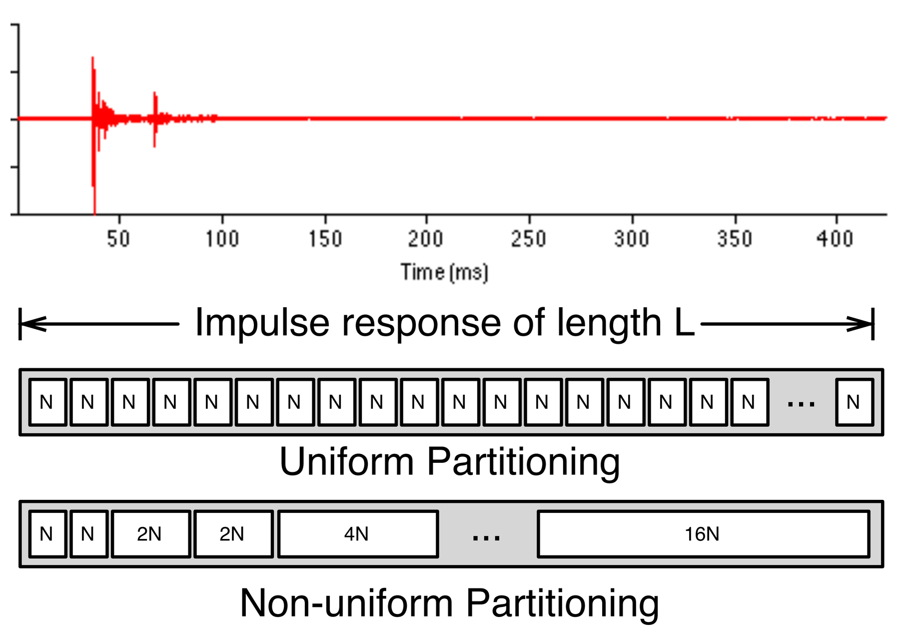
Algorithm 1 SNUP-MDF Echo Canceller

Input: microphone signal segment $\mathbf{d}(m)$,
loudspeaker signal segment $\mathbf{x}(m)$
Output: residual echo $\hat{\mathbf{e}}(m)$
$m \leftarrow 0$
$\mathbf{G}_{1j} = \begin{bmatrix} \mathbf{0}_{N\times N} & \dots & \dots & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \dots & \dots & \mathbf{I}_{N\times N} \end{bmatrix} \mathbf{F}^{-1}_{2B_jN}$
$\mathbf{G}_{2j} = \mathbf{F}_{2B_jN} \begin{bmatrix} \mathbf{I}_{B_jN\times B_jN} & \mathbf{0}_{B_jN\times B_jN} \\ \mathbf{0}_{B_jN\times B_jN} & \mathbf{0}_{B_jN\times B_jN} \end{bmatrix} \mathbf{F}^{-1}_{2B_jN}$
while halting criterion **do**
for $j \in J$ **do**
Construct J observation matrices from the loudspeaker signal
$B_T = \sum_{i=1}^{j} B_i$
$\mathbf{X}_j = \text{diag}\{\mathbf{F}_{2B_jN}[x((m - B_T - B_j)N),$
$\dots, x((m - B_T + B_j)N - 1)]^T\}$
end for
Modeled loudspeaker signal
$\hat{\mathbf{y}}(m) = \sum_{j=0}^{J-1} \mathbf{G}_{1j}\, \mathbf{X}_j(m)\hat{\mathbf{h}}_j(m)$
Microphone signal
$\mathbf{d}_0(m) = \mathbf{F}_{2N}[\mathbf{0}_{1\times N}, d(mN), \dots d(mN + N - 1])^T$
Residual echo
$\mathbf{e}_0(m) = \mathbf{d}_0(m) - \hat{\mathbf{y}}(m)$
$\mathbf{e}(t, m) =$ last N terms of $\mathbf{F}^{-1}_{2N}\mathbf{e}_0(m)$
for $j \in J$ **do**
$\mathbf{e}_j(m) = \mathbf{F}_{2B_jN}\left[\mathbf{0}_{1\times B_jN}, \mathbf{e}_0(m), \dots, \mathbf{e}_0(m)\right]^T$
Power spectrum estimate
$\mathbf{P}_{\mathbf{X}_j\mathbf{X}_j}(m) = \beta\mathbf{P}_{\mathbf{X}_j\mathbf{X}_j}(m) + (1 - \beta)\mathbf{X}_j^H(m)\mathbf{X}_j(m)$
Gradient calculation
$\boldsymbol{\nabla}\hat{\mathbf{h}}_j(m) = \mathbf{P}^{-1}_{\mathbf{X}_j\mathbf{X}_j}(m)\mathbf{X}_j^H(m)\mathbf{e}_j(m)$
Update filter
$\hat{\mathbf{h}}_j(m+1) = \hat{\mathbf{h}}_j(m) + \mathbf{G}_{2j}\mu_0\boldsymbol{\nabla}\hat{\mathbf{h}}_j(m)$
Thresholding
$\hat{\mathbf{h}}_j(m+1) = T(\hat{\mathbf{h}}_j(m+1))$
end for
$m \leftarrow m + 1$
end while

1 Nonuniformly partitioned MDF

- MDF filter composed of J nonuniform partitions of B_jN samples.
- We define the vector of the length of the partitions normalized by N as:

$$\mathbf{B} = [B_0, B_1, B_2, \dots, B_{J-1}], \quad \text{and} \quad B_0 = 1.$$



Example of uniform and nonuniform partitioning for MDF.

2 Adaptation with a Sparse Criterion

- The optimization problem at frame m, for each j^{th} component is

$$\min_{\mathbf{h}_j(m+1)\in\mathbb{R}^{B_jN}} \|\mathbf{e}_j(m)\|_2^2.$$

- A second-order Taylor approximation of the cost function around the neighborhood of $\hat{\mathbf{h}}_j(m)$ determines the next update point on the error curve.
- Assuming that the whole partition vector $\mathbf{h}$ is sparse, we add a penalization term to the original problem (5):

$$\min_{\mathbf{h}_j(m+1)\in\mathbb{R}^{B_jN}} \|\mathbf{e}_j(m)\|_2^2 + \gamma_j\|\mathbf{h}_j(m+1)\|_1,$$

where γ_j controls *how sparse* the j^{th} filter should be.

- Problem can be solved efficiently using a Iterative Shrinkage-Thresholding Algorithm (ISTA): the 2-norm gradient problem is solved then followed by a shrinkage/thresholding step (6).
- ISTA generally uses a first-order gradient method, we use a more precise second-order approximation of the Taylor series where the Hessian is approximated by $\boldsymbol{P}^{-1}_{\mathbf{X}_k\mathbf{X}_k}(m)$. The filter update step of our algorithm then becomes

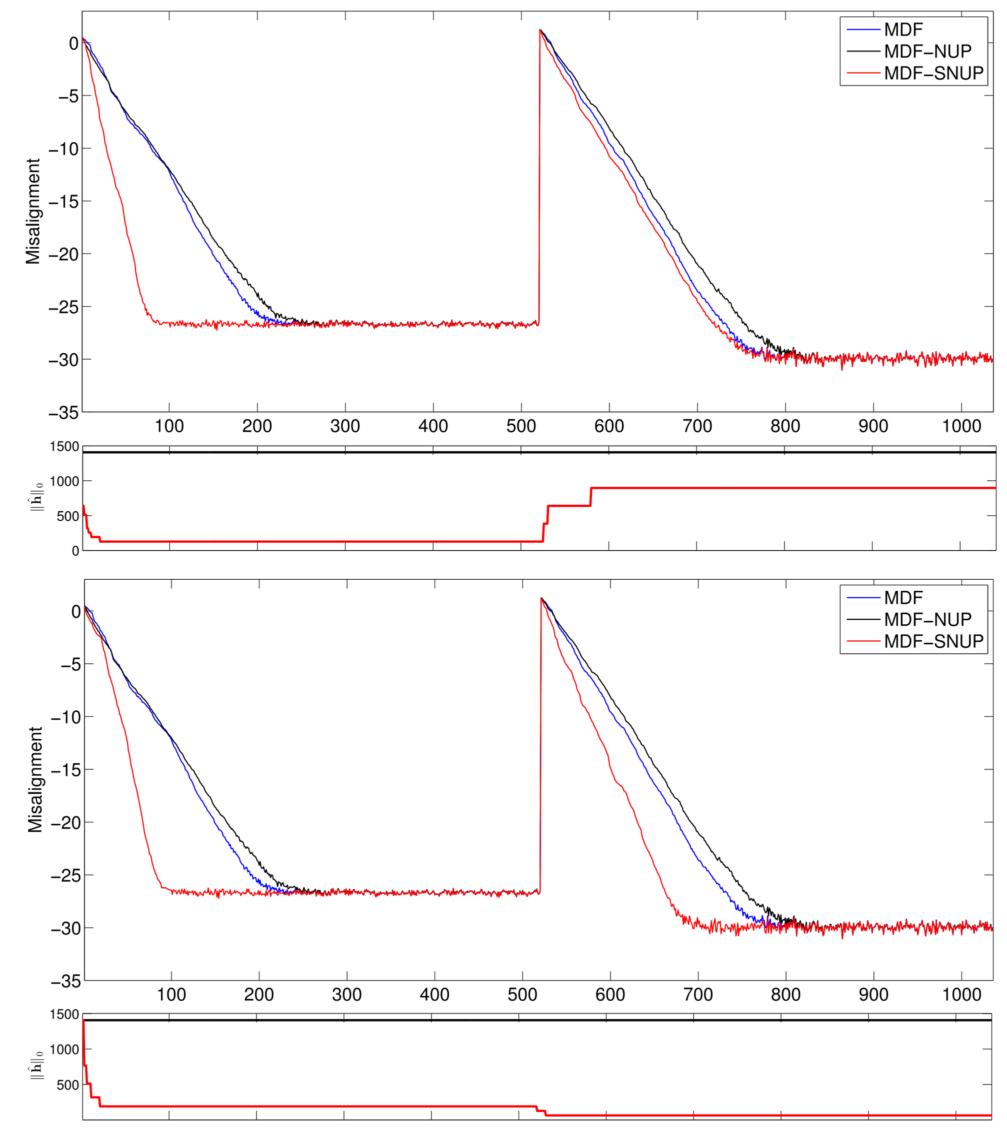
$$\forall j: \quad \hat{\mathbf{h}}_j(m+1) = T_\varepsilon\left(\hat{\mathbf{h}}_j(m) + \mathbf{G}_{2j}\mu_0\boldsymbol{\nabla}\hat{\mathbf{h}}_j(m)\right).$$

- We employ a hard-thresholding operator to distinguish between active and inactive partitions:

$$T_\varepsilon(\mathbf{h}_j) = \begin{cases} 0, & \|\mathbf{h}_j\|_1 \le \varepsilon_j; \\ \mathbf{h}_j, & \|\mathbf{h}_j\|_1 > \varepsilon_j. \end{cases}$$

- The choice of ε_j is not trivial, it should be proportional to the background noise floor $\varepsilon_j \propto \sigma_N$.
- Theory of optimality of soft-thresholding for sparse approximation carries over to hard-thresholding (6,7).

3 Experimental Results



Misalignment comparison for the changing impulse response sparsity. In the first setup, the true impulse response is initially very sparse, then changes into a significantly denser response $\xi(\mathbf{h}_1) \approx 0.12 \to \xi(\mathbf{h}_2) \approx 0.67$. In the second setup, we used two similarly sparse responses, a denser one first and a sparser one later $\xi(\mathbf{h}_1) \approx 0.19 \to \xi(\mathbf{h}_2) \approx 0.09$.
$\xi(\mathbf{h}) = (L/(L - \sqrt{(L)}))(1 - \|\mathbf{h}\|_1/\|\mathbf{h}\|_2)$.

- Evaluating the convergence of the algorithm and its behavior in tracking changes in model length (sparsity).
- Comparison with MDF, nonuniformly partitioned MDF (NUP-MDF), and the proposed sparse extension (SNUP-MDF).
- We allocated a $L = 1408$ filter and defined $N = 32$, which determined $K = 44$ partitions for the uniform MDF. For the NUP-MDF ($\sum_j NB_j = 1408$): $\mathbf{B} = [1, 1, 1, 1, 2, 2, 2, 2, 4, 4, 4, 4, 8, 8]$.
- Downlink signal defined as $x(k) = 0.8x(k-1) + n(k)$, where $n(k) \propto N(0, 1)$.
- The signal is then convolved with the true impulse response and white noise is added with SNR=25 dB.
- $\varepsilon_j \propto$ SNR, $\beta = 0.85$, $\mu = 0.2$.

4 Conclusions

- **Sparsity $\rightarrow$ intrinsic model order estimation.**
- Improvement in convergence speed.
- Potential savings in computation by avoiding multiplication, FFT, and IFFT of the unused partitions to calculate the output.
- Downside: complex architecture highly dependent on partition size choice. An efficient implementation of the NUP-MDF and SNUP-MDF is not trivial (8).

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